PARAMETRISATION OF THE DEFORMATION SPACE OF HYPERBOLIC POLYGONS VIA THE ARC COMPLEX

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Abstract: To every hyperbolic surface S, one can associate a simplicial complex, called the *arc and curve complex*, $AC(S)$, whose 0-th skeleton is the set of all isotopy classes of simple closed loops, that do not bound a disc or a punctured disc and embedded arcs with endpoints on the boundary of the surface and it has a k-dimensional simplex for each $(k+1)$-tuple of distinct disjoint isotopy classes. For a surface S with boundary with a complete finite-are hyperbolic metric, the projectivisation of the set of all disjoint union of geodesics along with a transverse measure, called the projective measured laminations of S, $\mathcal{PML}_{AC}(S)$, compactifies the arc and curve complex. We have proved that $\mathcal{PML}_{AC}(S)$ is homeomorphic to a sphere of dimension one less than that of the Teichmüller space $\mathcal{T}(S)$ of the space, generalising Thurston's result Theorem 5.1 in [2] on orientable hyperbolic surfaces without boundary. The arc complex $A(S)$ is a subcomplex of $AC(S)$, generated by the isotopy classes of only the embedded arcs. For certain surfaces such as hyperbolic polygons, that have finitely many arcs and no curves, the space $\mathcal{PML}_{AC}(S)$ coincides with the arc complex. The latter can be then used to completely parametrise the deformation spaces of both compact and non-compact polygons, using strip deformations. This is a partial generalisation of Theorem 1.5 in [1].

References

[1] Jeffrey Danciger, François Guéritaud, and Fanny Kassel. Margulis spacetimes via the arc complex. *arXiv:1407.5422 [math]*, July 2014. arXiv: 1407.5422.

[2] William P. Thurston. Minimal stretch maps between hyperbolic surfaces, 1986.