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AN AMBIENT APPROACH TO CONFORMAL GEODESICS

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Abstract:

Conformal geodesics are distinguished curves on a conformal manifold, loosely analogous to geodesics of Riemannian geometry. One definition of them is as solutions to a third order differential equation determined by the conformal structure. There is an alternative description via the tractor calculus. In this article we give a third description using ideas from holography. A conformal n-manifold X can be seen (formally at least) as the asymptotic boundary of a Poincaré–Einstein $(n+1)$-manifold M. We show that any curve γ in X has a uniquely determined extension to a surface Σ_γ in M, which which we call the *ambient surface of* γ. This surface meets the boundary X in right angles along γ and is singled out by the requirement that it it be a critical point of renormalised area. The conformal geometry of γ is encoded in the Riemannian geometry of Σ_γ. In particular, γ is a conformal geodesic precisely when Σ_γ is asymptotically totally geodesic, i.e. its second fundamental form vanishes to one order higher than expected.

We also relate this construction to tractors and the ambient metric construction of Fefferman and Graham. In the $(n+2)$-dimensional ambient manifold, the ambient surface is a graph over the bundle of scales. The tractor calculus then identifies with the usual tensor calculus along this surface. This gives an alternative compact proof of our ambient characterisation of conformal geodesics.