Forum des jeunes mathématiciennes
Forum of youngs mathematicians
October 23rd-25th 2019

– Plenary talks –

ALESSANDRA SARTI
University of Poitiers
K3 surfaces and their group of symmetries

Abstract : Particularly interesting objects in algebraic geometry are K3 surfaces, which are special complex algebraic surfaces. The easiest example of such a surface is the zero set of a homogeneous polynomial of degree four in the three dimensional complex projective space. The name was given by André Weil in 1958 in honor of three famous mathematicians : Kummer, Kähler and Kodaira and in honor of the K2 mountain at Cachemire. Their symmetry group is an important tool to understand their geometry. I will first show some remarkable properties of K3 surfaces and in particular the important role of lattice theory, then I will show some classic and recent results on their symmetry groups.

MARIE ALBENQUE
École Polytechnique
Scaling limit of random planar maps.

Abstract : In the sixties, Tutte introduced planar maps, that is the embedding of a planar graph in the sphere. He enumerated several families of maps and found beautiful and strikingly simple formulas. Forty years later, Schaeffer (following some previous work by Cori and Vauquelin) built some bijections to explain the simplicity of these formulas.

Thanks to this bijection, Chassaing and Schaeffer started studying random planar maps in 2004. Since this initial work, random planar maps have become a very active topic of research at the crossroads between probability theory, combinatorics and theoretical physics. In 2013, Miermont and Le Gall proved independently that the scaling limit of random quadrangulations is the so-called Brownian map.

In my talk, I'll present and explain the result of Miermont and Le Gall and will time some intuitions of the construction of the Brownian map by relying mostly on the combinatorial construction

behind their proof. I'll also give some extension of this result and the main open problems in this field. My talk won't require any prerequisite in probability theory or in combinatorics.

ANDA DEGERATU

University of Stuttgart

Gluing techniques for metrics

Abstract : Gluing techniques have been establishing themselves as a standard, albeit difficult, approach for obtaining geometrical objects with special properties. In this talk, I will focus on this technique in the context of Kähler metrics which are Ricci-flat, or more generally, have constant scalar curvature. The first requirements for its success are to have a large enough pool of model metrics and to figure out the obstructions for gluing them together into metrics with desired properties.

BEATRICE POZZETTI

University of Heidelberg

The critical exponent for higher rank Teichmuller spaces

Abstract : Higher rank Teichmuller spaces are unexpected connected components of the variety of homomorphisms of the fundamental group of a hyperbolic surface in a semisimple Lie group, that only consist of injective homomorphsims with discrete image. They thus generalize the Teichmuller space, and can be thought of as parametrizing certain locally symmetric spaces of infinite volume. After motivating the study of higher rank Teichmuller theories, I will discuss joint work with Andres Sambarino and Anna Wienhard in which we prove a sharp upper bound for the exponential orbit growth rate of the associated actions on the symmetric space.

– Short talks –

Wednesday, October 23th

Amphitéâtre Hermite

IRENE PASQUINELLI
Sorbonne Université
Deligne-Mostow lattices and cone metrics on the sphere

Key words : Complex hyperbolic geometry, lattices in complex Lie groups, geometric structures on low-dimensional manifolds.
Abstract : Finding lattices in $PU(n,1)$ has been one of the major challenges of the last decades. One way of constructing a lattice is to give a fundamental domain for its action on the complex hyperbolic space. One approach, successful for some lattices, consists of seeing the complex hyperbolic space as the configuration space of cone metrics on the sphere and of studying the action of some maps exchanging the cone points with same cone angle. In this talk we will see how this construction can be used to build funda- mental polyhedra for all Deligne-Mostow lattices in $PU(2,1)$.

FRANCESCA CAROCCI
University of Edinburgh
Donaldson-Thomas invariants and flopping curves

Key words : Algebraic geometry, Enumerative geometry, Moduli spaces
Abstract : The Donaldson-Thomas invariants counting 1 dimensional semistable sheaves on a Calabi-Yau 3-folds are closely related to the BPS invariants coming from physics, namely the fundamental integer valued in- variants that conjecturally underly all of the counting theories. These DT invariants are expected to satisfy a series of remarkable identities, among which the multiple cover formula. I will recall the proof of the multiple cover formula conjecture in the known cases and explain why the argument breaks in general . I will then concen- trate on flopping curves. In this case it is possible to reformulate the problem using quiver representations, and upgrade the conjecture from an equality of numbers to an isomorphism of vector spaces. Finally, I will explain how in this setting a certain cohomological operator plays as substitute to the main ingredient used in the proof of the known cases, and conclude with an update on what we have proved so far using our operator. *Joint with* Sjoerd Beentjes.

NICOLETTA TARDINI
University of Torino
Variational problems in conformal geometry

Key words : Complex manifolds ; special Hermitian metrics ; conformal
Abstract : Abstract : On a complex manifold one can define special classes of Hermit- ian metrics by imposing that the associated fundamental form is in the kernel of a special operator. One could expect that these metrics arise as critical points of naturally defined functionals on the space of Hermitian metrics. We will investigate some of these functionals, restricted to a conformal class of normalized Hermitian metrics, discussing the geometric meaning of their critical points. This is a joint work with Daniele Angella, Nicolina Istrati and Alexandra Otiman.

Salle 314

MÉLANIE THELLIÈRE
Institut Camille Jordan, Lyon
Corrugations and C^1-fractalty

Key words : differential geometry, convex integration, isometries.
Abstract : Convex Integration is a general method developped by Mikhaïl Gromov to solve differential relations. It has been used in 2012 to produce the first images of an isometric embedding of the flat torus into the 3-dimensional Euclidean space. These images reveal a geometric structure that is simultaneously C^1 and fractal. In this talk, we address the general question of the appearance of the C^1-fractality in the solutions built by Convex Integration. To do so, we first define the notion of Kuiper differential relations. We then show that such relations allow the emergence of a C 1 fractal behavior. We illustrate this point by considering totally real isometric maps : we first state a C^1 isometric embedding theorem "à la Nash-Kuiper" and then show that totally real isometric maps obtained by Convex Integration have C^1-fractal properties.

ALESSANDRA PLUDA
Università di Pisa
Calibrations for minimal Steiner networks

Key words : Steiner problem, covering spaces, calibrations.
Abstract : The Steiner problem in its classical formulation reads as follows : given a finite collection of points S in the plane, find the connected set that contains S with minimal length. Although existence and regularity of minimizers is well known, in general finding explicitly a solution is extremely challenging, even numerically. For this reason every method to determine solutions is welcome. A possible tool is the notion of calibrations. In this talk I will define calibrations for the Steiner problem within the framework of covering spaces. I will also give some example of both existence and non-existence of calibrations and to overcome this second unlucky case I will introduce the notion of calibration in families.

POLYXENI SPILIOTI
University of Tübingen
Dynamical zeta functions, Fried's conjecure and refined analytic torsion

Abstract : The dynamical zeta functions of Ruelle and Selberg are func- tions of a complex variable s and are associated with the geodesic flow on the unit sphere bundle of a compact hyperbolic manifold. Their representation by Euler-type products traces back to the Riemann zeta function. In this talk, we will present trace formulae and the machinery that they provide to study the analytic properties of the dynamical zeta functions and their relation to the analytic torsion, a spectral invariant. One can refer to this relation as the so called Fired 's conjecture. In the case of a non-unitary twist, i.e., a non-unitary representation of the fundamental group of the manifold, one has to consider a refinement of the analytic torsion as it is introduced by Braverman and Kappeler. In addition, time depending, we will present a comparison of the refined analytic torsion and the Cappell-Miller torsion.

– Panel discussion : careers of mathematicians –

Amphitéâtre Hermite

David Bessis, founder and CEO, Tinyclue, previously CNRS researcher.

Adriane Kaïchouh, professor at Lycée Pierre-Gilles de Gennes.

Vincent Despiegel, Research Team Leader in Machine Learning (Object Detection and Tracking), IDEMIA.

Pooran Memari, CNRS Researcher.

Jasmin Raissy, Assistant Professor at Université Paul Sabatier, Toulouse.

Alessandra Sarti, Professor at Université de Poitiers.

Thursday, October 24th

Amphitéâtre Darboux, morning session

ANNE LONJOU
University of Basel
Elements generating a proper normal subgroup in the Cremona group

Key words : Birational geometry, small cancellation, hyperbolic spaces.
Abstract : A birational transformation of the projective plane is an isomorphism between two dense open subsets of the projective plane. We are interested in the Cremona group, namely the group of birational transfor- mations of the projective plane. A key tool to study this group is its action on a infinite dimensional hyper- bolic space. Using this action, S. Lamy and S. Cantat answered positively, when the field is algebraically closed, the following long standing open question : Is the Cremona group simple ? During my phd, I extended this result to any field by finding an element satisfying the WPD (weakly properly discontinous) property. This implies by a work of F. Dahmani, V. Guirardel and D. Osin that the normal subgroup generated by a power of this element is a proper subgroup of the Cremona group. The question is then, which kind of elements generates a proper normal subgroup of the Cremona group ? We answered this question in a work in collaboration with S. Cantat and V. Guirardel.

ELSA GHANDOUR
Lund University
Almost complex surfaces in the nearly Kähler $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$

Key words : submanifolds, Nearly Kähler structure, $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$.

Abstract : This work is in the domain of differential geometry of submanifolds. We show that there exists a Nearly Kähler structure on the product $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ where $\mathrm{SL}(2,\mathbb{R})$ is the set of real square 2×2-matrices, which are of determinant 1. We then investigate almost complex submanifolds in this space, which means the submanifolds of $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ whose tangent space is invariant under the almost complex structure.

CLAIRE BRÉCHETEAU
École Centrale de Nantes
Robust shape inference from a sparse approximation of the gaussian trimmed loglikelihood

Key words : Geometric inference, Robustness, Support estimation
Abstract : Given a noisy sample of points lying around some shape outliers or clutter noise, we focus on the question of M, with possibly recovering M. Often, such inference is based on the sublevel sets of distance-like functions such as the function distance to M, the distance-to-measure (DTM) or the k- witnessed distance.

A sparse approximation of the DTM, the m-power-distance-to-measure (m-PDTM) is introduced and studied. Its sublevel sets are unions of m balls, with m possibly much smaller than the sample size. By miming the construction of the m -PDTM from the DTM, we propose an approximation of the trimmed log-likelihood associated to the family of Gaussian distribu- tions on $\mathbb{R}^d$. Its sublevel sets are unions of m ellipsoids.

We provide Lloyd-type algorithms to compute the centers of the balls and ellipsoids. Trimmed versions of these algorithms allow to wipe out clutter noise and to recover the homology of M from noisy data ; this requiring the storage of only m points and covariance matrices.

Joint with Clément Levrard

Salle 314, morning session

VIOLA SICONOLFI

Università di Roma Tor Vergata

Coxeter groups, Bruhat graphs and Ricci curvature

Key words : Coxeter groups, Bruhat graphs, discrete Ricci Curvature.

Abstract : The notion of discrete Ricci curvature for graphs was introduced in 1999 by Schmuckenschlager, it is part of the attempt to translate some notions of Riemannian geometry to graph theory. Among some more recent approaches to the discrete Ricci curvature there is a paper from Klartag et al. where the curvature of various Cayley graphs is computed. From these results arose the interest for the discrete Ricci curvature of graphs in Coxeter theory. During the talk I will recall the definition of discrete Ricci curvature together with some basic properties. I will then compute it for graphs associated to Coxeter groups, namely Bruhat graphs and weak order graphs. Finally I will describe some general results about the discrete Ricci curvature.

CHIARA RIGONI

University of Bonn

Recognizing the flat torus among $RCD^*(0,N)$ spaces via the study of the first cohomology group

Key words : Analysis and Geometry on metric measure spaces, Riemannian Geometry, and Optimal Transport Theory.

Abstract : A classical result due to Bochner says that for a compact, smooth and connected Riemannian manifold with non-negative Ricci curvature, the dimension of the first cohomology group is bounded from above by the dimension of the manifold. Moreover if these two dimensions are equal, then the manifold is the flat torus. In this talk I present a generalization of this result to the non-smooth setting of `RCD` spaces, by proving that if the dimension of the first cohomology group of a $RCD^*(0,N)$ space is N, then it is possible to construct an isomorphism between the space and the N-dimensional torus, equipped with its Riemannian distance and a constant multiple of the induced volume measure.

LILIA MEHIDI
Institut de Mathématiques de Bordeaux
Lorentzian tori without conjugate points

Key words : Lorentzian surfaces, conjugate points, Jacobi equation.

Abstract : In this talk, I will be interested in the conjugate points of Lorentzian surfaces. The absence of conjugate points in the Riemannian setting has rigid effects on the topology of the variety, and even on the metric structure. A result by E. Hopf (1948) states that any Riemannian metric on the torus T^2 with no conjugate points is necessarily flat. However, it appears that the Hopf theorem does not hold in the Lorentzian setting : there exists a non-flat Lorentzian torus without conjugate points, called the Clifton-Pohl torus (Bavard-Mounoud, 2013). There are natural ways to deform the Clifton-Pohl metric into metrics without conjugate points, but these tori are all modeled, up to projective equivalence, on the same universal object ; we say that they have (projectively) the same "local geometry". In this talk, we will explain that there exists from the point of view of local geometry infinitely many examples of Lorentzian tori without conjugate points.

Amphitéâtre Darboux, afternoon session

CATERINA VÂLCU
Ecole Polytechnique
Initial data mappings in general relativity

Key words : general relativity, conformal method, asymptotic analysis.
Abstract : We study initial data in general relativity, which are defined as solutions to the constraint equations. The focus in this talk is a modified version of the conformal method pro- posed by David Maxwell. While the model seems more strongly justified from a geometrical standpoint, the resulting system becomes significantly more difficult to solve ; it presents critical nonlinear terms, including gradi- ent terms. We work in dimensions 3,4 and 5, under smallness assumptions and in the presence of a scalar field with positive potential. The tools we use are related to obtaining a priori estimates (compactness results).

YANNICK HERFRAY
Université Libre de Bruxelles
An ambient approach to conformal geodesics

Key words : Conformal Geometry, Conformal Geodesics, Fefferman- Graham expansion, Tractor calculus
Abstract : Conformal geodesics are distinguished curves on a conformal manifold, loosely analogous to geodesics of Riemannian geometry. One definition of them is as solutions to a third order differential equation determined by the conformal structure. There is an alternative description via the tractor cal- culus. In this article we give a third description using ideas from holography. A

conformal n -manifold X can be seen (formally at least) as the asymptotic boundary of a Poincaré–Einstein $(n+1)$-manifold M. We show that any curve γ in X has a uniquely determined extension to a surface Σ_γ in M , which which we call the ambient surface of γ. This surface meets the boundary X in right angles along γ and is singled out by the requirement that it it be a critical point of renormalised area. The conformal geometry of γ is encoded in the Riemannian geometry of Σ_γ. In particular, γ is a conformal geodesic precisely when Σ_γ is asymptotically totally geodesic, i.e. its second fundamental form vanishes to one order higher than expected. We also relate this construction to tractors and the ambient metric construction of Fefferman and Graham. In the $(n+2)$-dimensional ambient manifold, the ambient surface is a graph over the bundle of scales. The tractor calculus then identifies with the usual tensor calculus along this surface. This gives an alternative compact proof of our ambient characterisation of conformal geodesics. *Joint with* Joel Fine.

Salle 314, afternoon session

DILETTA MARTINELLI
Universiteit van Amsterdam
Boundedness results in birational geometry

Key words : Birational geometry, Moduli spaces, Minimal Model Program
Abstract : The Minimal Model Program (MMP) is one of the main tools in the classification of higher dimensional algebraic varieties. Given a projective variety X, the goal of the program is to produce, with elementary surgeries, a new variety Y with better properties. We call Y a minimal model for X. After it is established that a minimal model exists, it is natural to ask whether this minimal model is unique, and if not how many there are and how do they behaved in familes. I plan to give a gentle introduction to the main ideas in the MMP focusing on the previous questions.

ALEXANDRA OTIMAN
Università Roma Tre
On a class of Kato manifolds

Abstract : In this talk we describe Kato manifolds, also known as manifolds with global spherical shell. Following a construction of M. Brunella, we prove that a large class of these manifolds carries locally conformally Kähler metrics. We then consider a specific class, which can be seen as a higher dimensional analogue of Inoue-Hirzebruch surfaces, and study several of their analytical properties. In particular, we give new examples, in any complex dimension $n \geq 3$, of compact locally conformally Kähler manifolds with algebraic dimension $n-2$, algebraic reduction bimeromorphic to $\mathbb{CP}^{n-2}$ and admitting non-trivial holomorphic vector fields. These results are joint work with Nicolina Istrati (University of Tel Aviv) and Massimiliano Pontecorvo (Roma Tre University).

–Gender equality talk –

ARTEMISA FLORES ESPINOLA
Centre de Recherches Sociologiques et Politiques de Paris
Femmes, genre et sciences : déchiffrer les inégalités, équilibrer l'équation

Friday, October 25th

Amphitéâtre Darboux

ANNA SIFFERT

Max Planck Institute for Mathematics

Construction of harmonic maps

Key words : Harmonic maps, singular ODE, symmetry.

Abstract : Geometric variational problems frequently lead to analytically extremely hard, non-linear partial differential equations, where the stan- dard methods fail. Thus finding non- trivial solutions is challenging. The idea is to study solutions with a certain minimum level of symmetry (i.e. group actions with low cohomogeneity), and use the symmetry to reduce the original problem to systems of non-linear ordinary differential equations, typically with singular boundary values. In my talk I explain how to con- struct harmonic mappings between manifolds with a lot of symmetry (i.e. cohomogeneity one manifolds). If time permits, I will discuss applications of the developed methods.

VALENTINA FRANCESCHI

Sorbonne Université

Sub-Riemannian soap bubbles

Key words : sub-Riemannian geometry, isoperimetric problem, minimal bubble clusters.

Abstract : The aim of this seminar is to present some results about minimal bubble clusters in some sub-Riemannian spaces. This amounts to finding the best configuration of $m \in \mathbb{N}$ regions in a sub-Riemannian manifold enclosing given volumes, in order to minimize their total perimeter. In a n-dimensional sub-Riemannian manifold, the perimeter is a non-isotropic $(n-1)$-dimensional measure that is defined according to the geometry. After an introduction to the subject, we will present some results concerning the cases $m = 1$ (isoperimetric problem) and $m = 2$ (double bubble problem), in a class of sub-Riemannian structures connected to the Heisenberg geometry. This is the framework of an open problem about the shape of isoperimetric sets, known as Pansu's conjecture. The results that will be presented are based on joint works with Roberto Monti (University of Padova), Aldo Pratelli (University of Pisa) and Giorgio Stefani (SNS, Pisa).

PALLAVI PANDA

Université de Lille

Parametrisation of the deformation space in hyperbolic polygons via the arc complex

Key words : arc complex, hyperbolic polygons, deformation space

Abstract : To every hyperbolic surface S, one can associate a simplicial complex, called the arc and curve complex, $AC(S)$, whose 0-th skeleton is the set of all isotopy classes of simple closed loops, that do not bound a disc or a punctured disc and embedded arcs with endpoints on the boundary of the surface and it has a k-dimensional simplex for each $(k+1)$-tuple of distinct disjoint

isotopy classes. For a surface S with boundary with a complete finite-are hyperbolic metric, the projectivisation of the set of all disjoint union of geodesics along with a transverse measure, called the projective measured laminations of S, $\mathcal{PML}_{AC}(S)$, compactifies the arc and curve complex. We have proved that $\mathcal{PML}_{AC}(S)$ is homeomorphic to a sphere of dimension one less than that of the Teichmüller space $\mathcal{T}(S)$ of the space, generalising Thurston's result Theorem 5.1 in [2] on orientable hyperbolic surfaces without boundary. The arc complex $A(S)$ is a subcomplex of $AC(S)$, generated by the isotopy classes of only the embedded arcs. For certain surfaces such as hyperbolic polygons, that have finitely many arcs and no curves, the space $\mathcal{PML}_{AC}(S)$ coincides with the arc complex. The latter can be then used to completely parametrise the deformation spaces of both compact and non-compact polygons, using strip deformations. This is a partial generalisation of Theorem 1.5 in [1].

References
[1] Jeffrey Danciger, François Guéritaud, and Fanny Kassel. Margulis spacetimes via the arc complex. arXiv :1407.5422 [math], July 2014. arXiv : 1407.5422.
[2] William P. Thurston. Minimal stretch maps between hyperbolic surfaces, 1986.

Salle 314

LAURA FEDELE
IMJ-PRG

Deformations of Grothendieck rings ans cluster algebras

Key words : Representation theory, quantum affine algebras, quantum cluster algebras.

Abstract : The problem of describing the q-characters of all simple representations of the quantum affine algebra $U_q(\hat{g})$ was solved by Nakajima by considering a certain one-parameter deformation $\mathcal{K}_t$ of the Grothendieck ring associated to the category of finite-dimentional representations of $U_q(\hat{g})$. $\mathcal{K}_t$ has geometric origin, it was in fact introduced for simply-laced Lie algebras by works of Nakajima and Varagnolo-Vasserot in terms of perverse sheaves on quiver varieties. On the other hand, it is known from works of Hernandez-Leclerc, Bittmann, and Qin that it corresponds to a quantization of a cluster algebra structure on the classical Grothendieck ring. The algebraic construction of $\mathcal{K}_t$ given by Hernandez shows how additional quantum parameters can naturally appear in the picture. We will then con- sider deformations of quantum Grothendieck rings with several parameters. Surprisingly, we observe that the dependance in the additional parameters also comes from a certain deformation of quantum cluster algebra structures. This is a joint work with David Hernandez.

ENRICA MAZZON
Imperial College London

Mirror symmetry : from physics to non-archimedean geometry

Abstract : Mirror symmetry is a fast-moving research area at the boundary between mathematics and theoretical physics. Originated from observations in string theory, it suggests that certain geometrical objects (complex Calabi-Yau manifolds) should come in pairs, in the sense that each of

them has a mirror partner and these two share interesting geometrical properties. In this talk I will introduce some geometrical ideas inspired by mirror symmetry. In particular, I will go through the main steps which relate mirror symmetry to non-archimedean geometry.

LIANA HEUBERGER
Loughborough University
Smoothing toric Fano threefolds

Key words : Fano varieties, Mirror Symmetry, Toric degenerations.
Abstract : Laurent Inversion (LI) is a smoothing construction designed to find mirror pairs in the Fano case. Given a Laurent polynomial f supported on a 3D Fano polytope P, let X_P be the associated toric Fano threefold. The general LI construction then embeds X_P inside an ambient toric variety F. If in addition X_P is a complete intersection defined by line bundles on F, taking a general section gives a variety X which degenerates to X_P. The goal is for X to be as smooth as possible. The principal motivation for these constructions is the following : there is a conjectured one-to-one correspondence between certain deformation families of Fano varieties and equivalence classes of polytopes. Using this informa- tion, one can state a precise version of the mirror theorem for Fano varieties. In the context above, this directly translates to f being the mirror of X.